Applications Of Number Theory In Engineering

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ABSTRACT

Number Theory is dedicated to concrete questions about integers, to place an emphasis on problem solving by students. When undertaking a first course in number theory, students enjoy actively engaging with the properties and relationships of numbers. The number theory is a branch of mathematics which is primarily dedicated to the studyofintegers.Thenumbertheory,assuch,islessappliedinengineeringcomparedto calculus, geometry, etc. The problem was that it could not be used directly in any application. But, the number theory, combined with the computational power of modern computers, gives interesting solutions to real-life problems. It has many uses in various fieldssuchascryptography,computing,numericalanalysisandsoon.Here,wefocuson the applications of the number theory about engineeringchallenges.

Key words: Number theory, engineering applications,Cryptography.

1. INTRODUCTION

Number theory, known as the queen of mathematics is the branch of mathematics that concerns aboutthepositiveintegers1,2,3,4,5whichareoftencallednaturalnumbersandtheirappealing properties. From antiquity, these natural numbers classified as odd numbers, even numbers, square numbers, prime numbers, Fibonacci numbers, triangular numbers, etc. Due to the dense of unsolved problems, number theory plays a significant role in mathematics. The recent classification of number theory depending upon the tools used to address the related problems is shown in the Figure1.

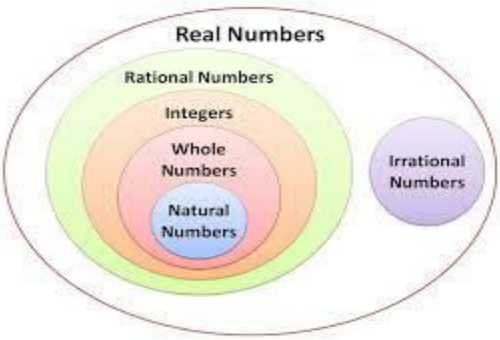

FIGURE1

Modern Classification of number theory as

1.Elementary Number Theory.

2.Algebric Number Theory.

3.Analytic Number Theory.

4.Geometric Number Theory.

5.Probablistic Number Theory.

TheresearchonintegersinascientificwayistrulycreditedtoGreeks.Later,abigrevolution onthistheoryhappened duetothearrivalofthefamousbook“Elements”byEuclidinwhichthe mathematics itself is depicted with preciseproofs.

There exist only a few k0 of literature discussing on the applications of number theory in engineering to the best of authors knowledge. So, the objective of present work is to perform a criticalreviewontheexistingpracticesrelatedtothenumbertheoryapplicationsinengineering.

2. APPLICATIONS OF NUMBERTHEORY

In the early period, number theory, a branch of pure mathematics was practically less applied in reallife.But,combinedwiththepresentcomputationtechnologies,itprovidessolutionstomany current problems. In this section, the authors discussed some of the applications of the theory related to engineeringtopics.

Congruence:A congruence is an equation in modular arithmetic, i.e., one in which only the remainders relative to some base, known as the "modulus," are significant.

Elliptic Curve:An elliptic curve is curve defined by an irreducible cubic polynomial in two variables.

Euclidean Algorithm: The Euclidean algorithm is an algorithm for finding the greatest common divisor of two numbers.

Cryptographyisoneoftheessentialfieldsintoday'sdigitalera,whereonlinesecurityisabig concern.Amessagesentfromasendertoreceiverinonlinecommunicationhastheriskofbeing seen by an unknown person without proper safety. This problem is solved by the use of the concept of encryption/decryption. The message which is sent by the sender is said to be ‘encrypted'orencodedwiththehelpofalargenumber,usuallyprime,whichissaidtobea‘key'; the receiver must have that same key to ‘decrypt' or decode the message. The application of number theory here is in the generation of such large prime numbers. Maurer devised an efficientalgorithmtogeneratesuchnumberswiththehelpofnumbertheory.Congruencemodulo relationisapartofmodulararithmetic,afundamentalpartofthenumbertheory.Thecongruence modulorelations,coupledwithlineartransformations,playsanessentialroleincryptography.A natural number expressed as the product of two prime numbers say $a_1a_2$where a_1, $a_2$need not bedistinctiscalledasemiprime.Semiprimesareexceedinglyhelpfulinthefieldofcryptography, most distinctly in public keycryptography.

‘Ellipticcurves’isanimportantconceptinnumbertheory.Investigationonnumbertheoretic queriesregardingellipticcurveswasformerlypursuedprimarilyforinventivereasons.Recently, these queries have become vital in many applied areas, including coding theory, pseudorandom number generation, and chiefly cryptography. There is even unique topic "ellipticcurve cryptography" in the field of cryptology. Coding theory, based on the number theory, serves as anadditionalshieldfortheexistingcrypticsystem.Pseudorandomnumbergenerationservesthe purposeof‘keys'verywell.SrikanthdiscussedthesuperellipticDiophantineequationwhich is an integral part of the field of number theory which is employed for many computer coding based applications.

There are many exciting series of numbers which have utmost importance for problem- solving. One such series is Fibonacci series (0, 1, 1, 2, 3, 5, 8…). It has multiple applications in engineering. In computer science engineering, the 'Fibonacci search technique,' as discussed by Ferguson, is a way of searching a sorted array. It uses a divide and conquers algorithm. This algorithm helps to narrow down the possible locations of the required element with the aid of Fibonacci numbers. Fibonacci search splits the array into two segments that have sizes that aresuccessiveFibonaccinumbers.Ithastheconveniencethatoneonlyneedstoperformaddition and subtraction to calculate the indices of the accessed array elements instead of other tedious operations. The time dependence of moments and size distributions during consolidation is the utilization of Fibonacci series in thesimulation.

Another significant concept related to the Fibonacci series is the golden ratio φ (phi). Any two quantities are said to be in the golden ratio if their ratio is equal to the ratio of their sum to the bigger of the two quantities. Represented algebraically, for two quantities x and y, $x > y > 0$, $(x + y)/x = x/y = \varphi$. Shapes of several natural and human-made objects are seen to obey the golden ratio. The spirals in the flowers of plants and Parthenon, the famous monument are some of the classic examples. Fibonacci series has found many uses in architecture as well as engineering and is widely seen in nature. The phi code explains the behavior of structural elements used in engineering. It is seen as a defining parameter in the stress analysis of beams. Collins and Brebbia pointed out the existence of phi code in the relation between normaland shear stresses. The normal stress σ_x and the maximum shear stress τ_{max}, for the condition $\sigma_x = \tau_{xy}$and $\sigma_y = 0$, is related as $\tau_{max} = \sigma_x [\sqrt{5} / 2]$, where $\sqrt{5} = (1 + \Phi^2)/\Phi$. It is a useful tool for structural analysis.

One of the famous theorems in Mathematics is the 'Pythagoras theorem.' It deals with the right-angled triangles, giving the relation between the sides. Unsurprisingly, it has applications in any field which deals with triangles. Few famous examples are listed next. The wing configuration used in modern jet aircraft is the 'Delta wing.' The theorem plays a part in the effective and efficient design of such configuration. Similar applications can be found in tips of rockets, which is an isosceles triangle in sectional view. Sectional analysis of frustum of cones, which serves as fairing between the stages of a multi-stage rocket, is also an example. Calculations of propeller and engine blade angles involve the theorem. Aerospace scientists and meteorologists find a range and sound source using this theorem. An interesting combination of non-arithmetic sequence and the Pythagoras theorem exists in the number theory. The sequence 3,5,9,11,15,19,21,25,29,35,consistsoflegsasoddnumbersinrighttriangleswiththelength of the sides being integers and hypotenuse length as a prime number.

The acoustic quality of concert halls can be improved with the help of number theory as discussed by Manfred. The construction of new musical scales to the optimum diffusion of soundinthehallsimprovestheacousticqualitytoagreatpracticalextent.Methodsforimproved sounddissipationbyreflectionphase-gratings,basedonthreedistinctconceptsofnumbertheory, are depicted by the work done by Manfred. Boris and Leonid discussed how the restricted partitionfunctioncouldbeemployedincomputingallalgebraicallyindependentinvariantsofthe degrees emerging from the action of the finite group on the vector space over the complex field. The use of restricted partition functions to the task of computing "algebraically independent invariants"ofthedegreeswhichemergeddueto anactionof"thefinitegroupinthevectorspace overthefieldofcomplexnumbers".Atwo-parametergeneralizationoftheentireellipticintegral ofthesecondkind,whichisgivenregardingtheAppellfunction,wasdiscussedbyVictorBarsan. This function is further reduced to a quite more comfortable bilinear form in the complete elliptic integrals, and a few real uses are shortly specified related to solid-state physics in this work.

New polynomial equivalents of Jacobi's triple product were given by KrishnaswamiAlladi and Alexander Berkovich . A simple introduction to both the mathematical and engineering forms of coding theory, weights of the codewords, were discussed by Robert and Howard. Roger presented actual characteristics of regular point lattices rising from a utilization-oriented perspective. He briefly revealed the characteristic of Farey sequences in plant biology. Equivalents of "Newton–Girard power-sum" formulae for whole and meromorphic functions with uses to the Riemann zeta function were discussed by Armen et al.

Finally, an application of Ramanujan sum in engineering is discussed. The form of this sum in signal processing was noticed over the past decades. Vaidyanathan showed how the Ramanujansumcouldimplementtopulloutperiodiccomponentsindiscretetimesignals.Again, Vaidyanathan introduced a subspace called Ramanujan subspace and studied its properties for proving the decomposition of finite duration signals into the finite sum of orthogonal subspaces.

Thus the extensive applications of number theory are noted in several areas. The current scenario is such that the role of number theory has more weight for cyber security problems. Futureapplicationsareabundantbecauseofthedevelopmentsinhigh-speedcomputers,andthere is a scope for the growth in the applications of numbertheory.

3. CONCLUSION

Various engineering applications of the number theory were mentioned in detail. Thesignificant contributionofnumbertheoryinrecentyearsisintheareaofcryptography,andhencecomputer scienceengineeringwasnotedinitially.Theimportanceoffamousseriesandsequencesinalmost every field of engineering was observed. It is seen that applications of number theory were not directly in some applications; with the number theory being fundamental, it acted as the driving forceinapproachingthesolution.Theversatilityoftheapplicationswasalsorecognized.Further researchanddevelopmentofthetheorywillpavethewayformoreusesofnumbertheorytoboth pure as well asapplied/engineeringmathematics.

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